The theoretical relationship for permanent enlargement ratio may be derived as follows. The enlargement of the bore under pressure is given by Eq. (21) substituting $r = a$. To find the permanent enlargement of the bore, the elastic recovery must be substracted from the enlargement under pressure. The elastic recovery is given by the Lamé equation and the value of pressure from Eq. (22).

The enlargement of the outside surface under internal pressure is found from Eq. (12), letting $r = b$ and the elastic recovery of the exterior surface is again subtracted as was that for the inside surface. Dividing the resulting equation for permanent bore enlargement by that for permanent enlargement of the outside surface yields the following equation for the permanent enlargement ratio.

$$PER = \frac{1}{2}\left[(2 - \mu)\, W + \frac{\mu}{W}\right] \qquad (25)$$

A plot of this equation is shown in Fig. 10 for $\mu = 0{\cdot}3$.

It is interesting to note that Eq. (25) can be derived directly from geometric considerations assuming no net change in volume as a result of overstrain.

Percent Permanent Bore Enlargement

From Fig. 9 it can be seen that, when a condition of 100 percent overstrain (strain factor = 1.0) is reached, the pressure-strain curve becomes essentially flat. It is, therefore, assumed that no additional benefit can be obtained by further deformation and that 100 percent overstrain is the optimum amount of deformation. Actually, in higher diameter ratios where reverse yielding will occur at the bore on the release of the 100 percent overstrain pressure, the optimum percent bore enlargement may be slightly less. However, in the diameter ratios investigated, this reverse yielding effect is considered negligible.

The percent permanent bore enlargement required to produce 100 percent overstrain was determined experimentally by plotting the maximum exterior surface strain factor vs. the percent permanent bore enlargement obtained for each cylinder. This yielded a family of curves, one for each diameter ratio. The point of intersection of each of these curves with the horizontal line representing a strain factor of 1.0 indicated the value of permanent bore enlargement to just produce 100 percent overstrain. These points are plotted in Fig. 11.

The theoretical curve shown is obtained by substituting $r = a$ and $\rho = b$ in Eq. (21) yielding the following equation for bore strain at 100 percent overstrain pressure.

$$\epsilon_{ta} = \frac{\sigma_y}{E}\left[-1.08(1 - 2\mu)\ln W + \frac{\mu}{2} + \left(\frac{2-\mu}{2}\right) W^2\right] \qquad (26)$$

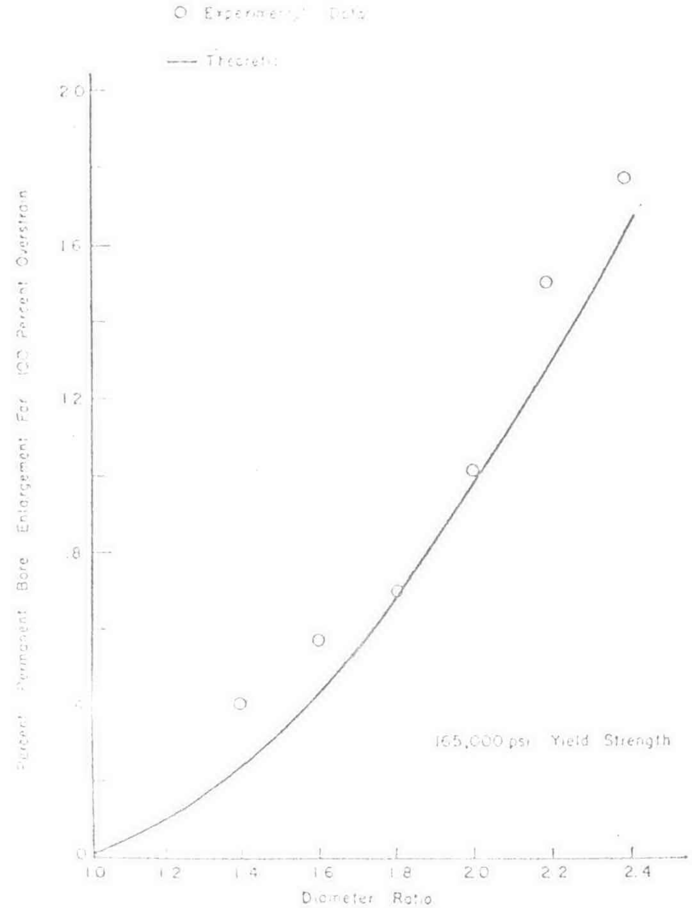

FIG. 11. Percent permanent bore enlargement for 100 per cent overstrain vs. diameter ratio.

The elastic recovery at the bore is given by the Lamé equations and Eq. (4).

$$\epsilon_{tae} = \frac{\sigma_y}{E}\,\frac{1.08\ln W}{W^2 - 1}\,[(1 + \mu)\, W^2 + (1 - \mu)] \qquad (27)$$

Subtracting Eq. (27) from Eq. (26) yields:

$$\epsilon_{ta\,perm} = [\mu + (2 - \mu) W^2]\left[\frac{1}{2} - \frac{1.08\ln W}{W^2 - 1}\right] \qquad (28)$$

This equation is plotted in Fig. 11 for $\mu = 0.3$ and is in good agreement with the experimental values.